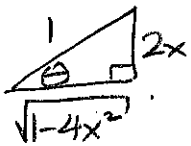
Simplify the following expressions completely. Show proper reasoning to justify your answer. **SCORE: ____ / 12 PTS**

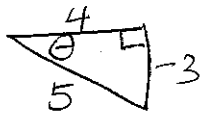
[a] $\cot(\sin^{-1} 2x)$



$\sin\theta = 2x$

$\cot\theta = \frac{\sqrt{1-4x^2}}{2x}$

[b] $\csc(\arctan(-\frac{3}{4}))$



$\tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3}$

Graph 2 periods of the function $y = -3\cos(\frac{3\pi}{2}x + \frac{7\pi}{3}) - 2$. **SCORE: ____ / 16 PTS**

Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete periods, starting at the phase shift.
Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.

MIDDLE $= -2$

AMPLITUDE $= |-3| = 3$

MAX $= -2+3 = 1$

MIN $= -2-3 = -5$

$-\cos$ 

PERIOD $= \frac{2\pi}{\frac{3\pi}{2}} = 2\pi \cdot \frac{2}{3\pi} = \frac{4}{3}$

$\frac{1}{4}$ PERIOD $= \frac{1}{3}$

PHASE SHIFT $\frac{3\pi}{2}x + \frac{7\pi}{3} = 0$

$x = -\frac{7\pi}{3} \cdot \frac{2}{3\pi} = \frac{-14}{9}$

POINTS:

($\frac{-14}{9}$, -5)

($\frac{-11}{9}$, -2)

($\frac{-8}{9}$, 1)

($\frac{-5}{9}$, -2)

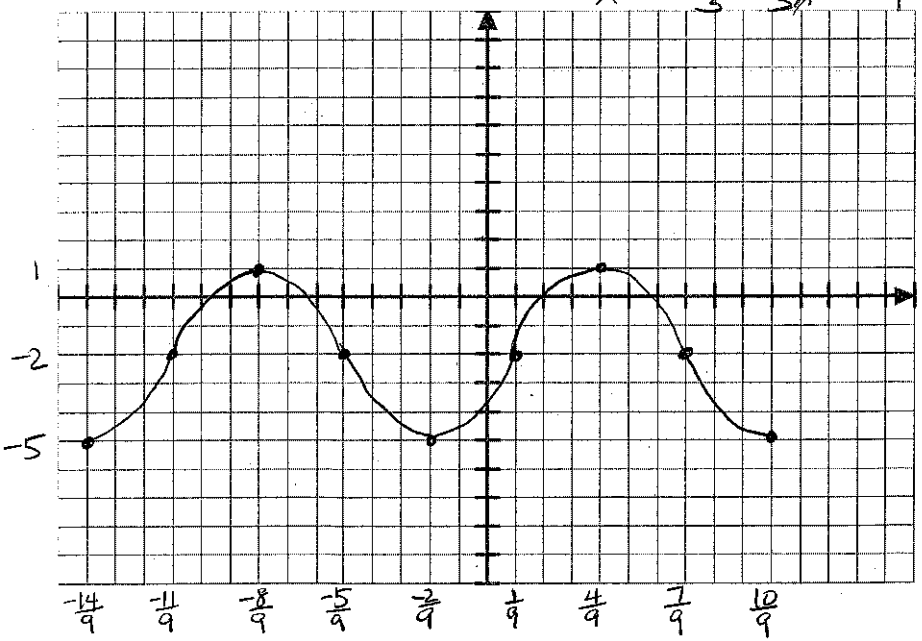
($\frac{-2}{9}$, -5)

($\frac{1}{9}$, -2)

($\frac{4}{9}$, 1)

($\frac{7}{9}$, -2)

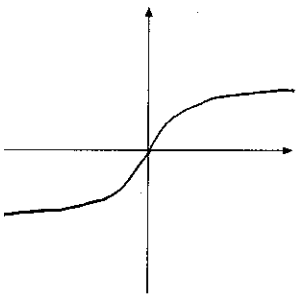
($\frac{10}{9}$, -5)



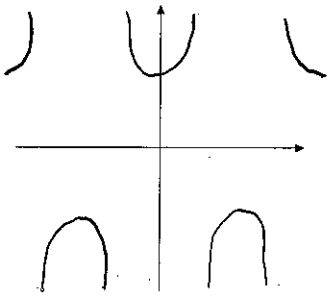
Sketch the graphs. **For periodic functions, sketch at least 2 periods.**
You only need to get the general position and shape correct. Do NOT plot points.

SCORE: ____ / 12 PTS

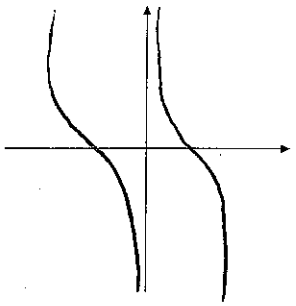
[a] $y = \tan^{-1} x$



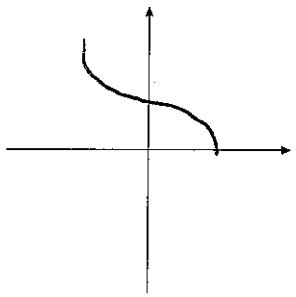
[b] $y = \sec x$



[c] $y = \cot x$



[d] $y = \cos^{-1} x$



Fill in the blanks. Write "DNE" if the question has no answer.

SCORE: ____ / 32 PTS

[a] The domain of $f(x) = \arcsin x$ is $[-1, 1]$.

[b] The range of $f(x) = \sec x$ is $(-\infty, -1] \cup [1, \infty)$

[c] The equations of the asymptotes of $f(x) = \tan^{-1} x$ are $y = \pm \frac{\pi}{2}$.

[d] The equations of the asymptotes of $f(x) = \csc x$ are $x = n\pi$.

[e] The range of $f(x) = \cos^{-1} x$ is $[0, \pi]$.

[f] The domain of $f(x) = \tan x$ is $x \neq \frac{\pi}{2} + n\pi$

[g] As $x \to \frac{\pi}{2}^+$, $\sec x \to -\infty$.

[h] As $x \to 0^-$, $\cot x \to -\infty$.

$\cos^{-1}\left(-\frac{1}{2}\right)$

[i] $\cos^{-1}(\cos\frac{4\pi}{3}) = \frac{2\pi}{3}$.

[j] $\tan(\tan^{-1} 2) = 2$.

[k] $\sin(\arcsin 2) =$ DNE.

[l] $\arccos(\cos\frac{3\pi}{4}) = \frac{3\pi}{4}$.

[m] $\tan^{-1}(-1) = \frac{-\pi}{4}$.

[n] $\arccos(-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$.

[o] $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

[p] $\arctan\sqrt{3} = \frac{\pi}{3}$.

BJ has unhealthy eating habits, which causes his weight to fluctuate up and down. **SCORE: _____ / 12 PTS**

On Jan 21st (the day he had 3 midterms back-to-back), his weight reached a peak of 177.3 pounds.

On Feb 7th (the day he took a mental health day and slept for 18 hours), his weight reached a low of 168.1 pounds.

[a] Assuming that BJ's weight corresponds to a sinusoidal function, find an equation for BJ's weight on the t^{th} day after Jan 1st.

MIDDLE $= \frac{1}{2}(177.3+168.1) = 172.7$

AMPLITUDE $= \frac{1}{2}(177.3-168.1) = 4.6$

PERIOD $= 2(37-20) = 34 = \frac{2\pi}{B} \rightarrow B = \frac{\pi}{17}$

JAN 21 $t=20$; FEB 7 $t=37$

$\frac{1}{2}$ PERIOD + COS

$$\text{WEIGHT} = 4.6\cos\tfrac{\pi}{17}(t-20)+172.7$$

[b] Based on the answer of [a], how much will BJ weigh on Feb 22nd ?

$t=52$

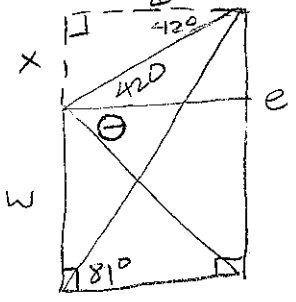
$$4.6\cos\tfrac{\pi}{17}(52-20)+172.7 = 177 \text{ POUNDS}$$

A 420 foot cable connects the roofs of two buildings. **SCORE: _____ / 16 PTS**

From the base of the west building, the angle of elevation to the roof of the east building is 81°.

From the roof of the east building, the angle of depression to the roof of the west building is 42°.

[a] Find the height of each building. (You may need to calculate other information that is not explicitly requested.)



$\cos 42^\circ = \frac{d}{420} \rightarrow d = 420\cos 42^\circ = 312$

$\sin 42^\circ = \frac{x}{420} \rightarrow x = 420\sin 42^\circ = 281$

$\tan 81^\circ = \frac{e}{312} \rightarrow e = 312\tan 81^\circ = 1970$

$w = 1970 - 281 = 1689$

EAST BUILDING = 1970 ft

WEST BUILDING = 1689 ft

[b] Find the angle of depression from the roof of the west building to the base of the east building.

$$\tan\theta = \frac{1689}{312} \rightarrow \theta = \tan^{-1}\frac{1689}{312} = 80^\circ$$